ABSTRACT: The lateral outflow mechanism of side weirs is investigated using a one-dimensional approach. In particular, the effects of flow depth, approaching velocity, lateral outflow direction and side weir channel shape are included, resulting in expressions for the lateral outflow angle and the lateral discharge intensity. For vanishing channel velocity, the conventional weir formula for plane flow conditions is reproduced. However, for increasing local Froude number, the lateral outflow intensity decreases under otherwise fixed flow parameters. A comparison of the theoretically determined solution with observations indicates a fair agreement. The results are then applied to open channel bifurcations by considering a side-weir of zero weir height. Distinction between sub- and supercritical flow conditions is made. Again, the computed results compare well with the observations and allow a simple prediction of all pertinent flow characteristics.

INTRODUCTION

Side weirs are structures often used in irrigation techniques, sewer networks and flood protection. The usual approach assumes one-dimensional flow conditions, thereby neglecting the transverse variations of the free surface profile and the velocity distribution. This investigation relates to spatially varied flow caused by the operation of side weirs, in which all flow parameters are gradually varied; in particular, hydraulic jumps are excluded.

The prediction of the one-dimensional flow characteristics (e.g., free surface profile and local distribution of discharge) is based upon the dynamical flow equation. Application of the longitudinal momentum theorem yields a generalized backwater relation as presented by Favre (2) and later extended by Yen and Wenzel (12). The dynamical flow equation must be completed by the lateral outflow law and expressions relating to the momentum correction coefficient, the lateral outflow velocity component in the longitudinal direction and the friction slope. The usual approach assumes a uniform velocity distribution, and is based on the energy equation, in which losses are accounted for by Manning's formula. Finally, the lateral outflow is modeled by the (eventually modified) weir equation valid for plane flow conditions. Detailed investigations regarding the flow characteristics in side weirs indicate that the last assumption is particularly unsuitable (3). Consequently, the lateral outflow from side weirs will be investigated in more detail herein.

The first systematic studies of the lateral outflow pattern must be credited to Coulomb, et al. (1). Therein, the average cross-sectional outflow direction is determined in terms of weir height and local velocity head $V^2/(2g)$ for broad-crested and sharp-crested weirs. However, no definite recommendations are given.

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Kosinsky (7) presents various propositions regarding the lateral outflow equation. However, all relations refer only to the flow depths at the up- and downstream bounding cross sections, and effects of the velocity of approach are incorporated by some experimental correction factors.

The first systematic investigation of the internal flow characteristics are explored by Subramanya and Awasthy (11) for a side weir of zero height. Applying Bernoulli's equation on a streamline leaving the channel laterally, the outflow direction and the discharge correction coefficient are determined in terms of the approaching Froude number. Two different propositions are given for sub- and supercritical inflow Froude numbers, the latter being adjusted by experimental means. In their discussion, Nadesamoorthy and Thomson (8) are able to correlate the data by an expression for arbitrary inflow Froude numbers. However, their approach overlooks the local flow pattern and relates only to a particular side weir configuration. Ranga Raju et al. (10) base their computational scheme on de Marchi's free surface profile, and adjust the discharge coefficient $Q$ in terms of the inflow Froude number $F_0$. For $F_0 \to 0$, the unrealistically high value $Q \to 0.521$ (instead of $C_d \to 0.42$) is extrapolated for sharp-crested weirs. For broad-crested weirs, a correction coefficient, depending on the ratio of head on the weir to crest length, is introduced. No results are given for supercritical side weir flow. Ramamurthy and Carballada (9) model the lateral outflow by an adaptation of the free streamline theory. However, the results apply only on particular weir geometries. Furthermore, the local flow characteristics are again overlooked. Consequently, backwater effects cannot be incorporated into the model equations.

The purpose of the present investigation is to analyze the lateral outflow pattern in side weirs. It will be shown that the plane lateral outflow over weirs ($F \to 0$) is a particular case of the side weir flow. One-dimensional relations for the average local outflow angle and the local outflow intensity are derived.

**PLANE WEIR FLOW**

The discharge characteristics of a plane, non-submerged weir in a rectangular channel are significantly influenced by the weir height $w$, the upstream flow depth $h$, and the weir crest shape. According to Poleni's formula

$$q = C_d \sqrt{2g(h - w)^{3/2}} \quad \quad \quad \quad \quad \quad (1)$$

in which $q$ is discharge per unit width; $g = \text{gravitational acceleration}$, and $(h - w) = \text{the pressure head on the weir}$. The discharge coefficient $C_d$ depends on the relative weir height $(h - w)/h$, and the crest shape, provided the effects of surface tension are excluded, $(h - w) > 0.03 \text{ m}$. In what follows, it is convenient to express $q$ as a function of the energy head $H = h + q^2/(2gh^2)$ on the weir [Fig. 1(a)], where

$$q = C_d \sqrt{2g(H - w)^{3/2}} \quad \quad \quad \quad \quad \quad (2)$$

It can be shown that $C_d$ then becomes much less sensitive to variations of $\chi = (H - w)/H$. The experimental results of Kandaswamy and Rouse
FIG. 1.—Plane Weir Flow: (a) Sharp-Crested; (b) Broad-Crested; (c) Round-Crested Weir Shape

(6) indicate for the sharp-crested, fully aerated thin-plate weir

\[ C_d = C_{d0} \left[ 1 + \left( \frac{x^3}{7} \right) \right]; \quad 0 \leq x \leq 1 \] .................................................... (3)

\[ C_{d0} = 0.42 \] is the discharge coefficient for \( x = 0 \).

Since the usual weirs have energy heads \( H < 2w \) (\( x < 1/2 \)) the effect of \( x \) on \( C_d \) is insignificant (+2%). With the exception of \( H/w > 2 \), \( C_d \) remains constant. For \( w = 0 \), thus \( x = 1/2 \), the correction factor \( c = C_d/C_{d0} = 8/7 \). For vertical broad-crested weirs with a crest length \( L \) [see Fig. 1(b)], the corresponding expression becomes \( C_d = cC_{d0} \), in which (3)

\[ c = 1 - \frac{2}{9(1 + \zeta_b^2)}; \quad \zeta_b = \frac{H - w}{L} \] ................................. (4)

The respective proposition of Ranga Raju, et al. (10) is \( c = 0.8[1 + \zeta_b/8] \). Finally, the weir shape coefficient \( c \) for the round-crested weir of crest radius \( r \) can be expressed as (3)

\[ c = \frac{\sqrt{3}}{2} \left( 1 + \frac{22}{81} \frac{r^2}{r^2} \right); \quad \zeta_r = \frac{H - w}{r} \] .................................................... (5)

Eq. 2 thus incorporates effects of the velocity of approach \( (H) \) and the weir crest geometry \( (c) \).

SIDE-WEIR

Compared to plane weir flow, streamlines over a side weir deviate from the channel axis by the angle \( \phi \). If the flow characteristics of the side weir are described by a one-dimensional approach, the energy head \( H \) with respect to channel bottom at a particular location is

\[ H = h + \frac{Q^2}{2gA^2} \] .................................................... (6)

when assuming hydrostatic pressure and uniform velocity distributions. \( Q \) is discharge; and \( A = \) cross-sectional area.
Let $Q' = dQ/dx < 0$ be the lateral outflow per unit length in which $x$ is the longitudinal coordinate. It is then convenient to express $Q'$ as

$$Q' = -\omega \cdot q \tag{7}$$

$q$ being the corresponding lateral outflow for plane flow conditions, Eq. 2. With $U$ as local lateral outflow velocity, the lateral outflow intensity becomes (see Fig. 2)

$$dQ = -U \cdot \sin \phi \cdot h \cdot dx \tag{8}$$

with $h$ as the typical depth of flow.

The lateral flow coefficient, $\omega$, thus includes effects of the lateral velocity, $U$, the lateral angle, $\phi$, and the lateral outflow depth $h$. To this end, it is possible to represent formally $\omega = \omega_u \cdot \omega_\phi \cdot \omega_h$. Evidently, for $\phi \to 90^\circ$ results $\omega \to 1$ (plane flow condition).

**Effect of Flow Depth.**—Fig. 3 compares the flow over a plane weir with the side weir. Note that the weir heights $w$ and flow depths $h$ are equal, whereas $H_1 < H_2$. It is seen that the velocity component perpendicular to the side weir axis vanishes at the side opposite to the outflow plane in case $b$. The flow depth $h$ then corresponds nearly to the energy head $H_1$ of plane weir flow provided the usual case $\sqrt{2gH} \to 0$ prevails. Since the transverse flow profile as shown in Fig. 3(b) is nearly horizontal, one may consider the flow depth $h$ in a side weir equal to the energy head $H = H_1$ of plane weir flow. The effect of $\omega_h$ thus transforms $q(H)$ as given in Eq. 2 into $Q'(h)$.

**Effect of Velocity of Approach.**—As shown in Fig. 3, two weirs of equal weir height $w$ and flow depth $h$ may have different velocities $V = Q/A$; correspondingly, the two energy heads $H_1$ and $H_2$ are different. Since $U_1 = \sqrt{2g(H_1 - w)}$ and $U_2 = \sqrt{2g(H_2 - w)}$, and with $H_1 \to h$ and

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**FIG. 2.—Lateral Outflow Geometry, Plan View; (a) Prismatic; (b) Non-Prismatic Side Weir; (c) Longitudinal; and (d) Transversal Sections**

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494
$H_2 \rightarrow H$, the effect of the approaching velocity is

$$\omega_u = \frac{U_2}{U_1} = \left[ \frac{(H - w)}{(h - w)} \right]^{1/2}$$

For plane flow conditions, $\omega_u(H/h \rightarrow 1) = 1$. However, $\omega_u > 1$ for side weir flow since $H/h > 1$. As a consequence, the velocity of approach augments the lateral outflow intensity $Q'$.

**Effect of Lateral Outflow Angle.**—According to Eq. 8 $\omega_\phi = \sin \phi$; the maximum of $\omega_\phi$ occurs for $\phi = 90^\circ$ (plane weir flow).

Side weirs are structures in which flow conditions vary gradually. Therefore, the head losses due to wall friction and lateral outflow are small (3). Approximately, potential flow conditions with a nearly uniform velocity distribution may be assumed. Accordingly, the average channel velocity $V$ and the axial component of the lateral outflow velocity $U \cdot \cos \phi$ are equal, $V = U \cdot \cos \phi$ (2). Expressing $\cos \phi = (1 - \sin^2 \phi)^{1/2}$ then yields

$$\omega_\phi = \left[ 1 - \left( \frac{V}{U} \right)^2 \right]^{1/2} \quad \text{(10)}$$

Fig. 2(d) shows a cross-sectional view of a side weir. Assuming essentially hydrostatic pressure and uniform velocity distributions for the channel flow, and pressure and velocity distributions as indicated at the lateral outflow zone, Bernoulli’s equation leads to

$$H = h + \frac{V^2}{2g} = w + \rho(h - w) + \frac{U^2}{2g} \quad \text{(11)}$$

in which $\rho$ is a residual pressure coefficient, $\rho < 1$.

Let

$$y = \frac{h}{H}; \quad W = \frac{w}{H} \quad \text{(12)}$$

then upon eliminating $\omega_\phi$ from $\omega_\phi = \sin \phi$ and Eq. 10

$$\sin \phi = \left[ \frac{(y - W)(1 - \rho)}{1 - \rho y - W(1 - \rho)} \right]^{1/2} \quad \text{(13)}$$
The actual pressure distribution at the weir crest is characterized by atmospheric pressure at the upper and lower nappe boundaries, the vertical flow depth being approximately $5(h - w)/6$ (6). Detailed computations indicate that $p = 2/3$ (3). Therefore, a substitute weir has hydrostatic pressure distribution with a vertical flow depth of $2(h - w)/3$. Inserting $p = 2/3$ into Eq. 13 yields

$$\sin \phi = \left[ \frac{y - W}{3 - 2y - W} \right]^{1/2} \quad \ldots \ldots \ldots \ldots \ldots (14)$$

Consequently, $\phi = 90^\circ$ for $y \to 1$ ($h = H$), which corresponds to negligible velocity of approach. For $y = W$ ($h = w$), no more water leaves the side weir laterally and $\phi = 0$. In contrast to the velocity of approach, the lateral outflow intensity $Q'$ is reduced for $\phi < 90^\circ$ when compared to the plane weir configuration.

**Effect of Channel Shape.—**Fig. 2(b) shows a plan view of a side weir with a longitudinal channel width contraction. According to Refs. 3 and 5, a width contraction yields a more uniform lateral outflow distribution and a nearly horizontal free surface profile. The other variant, a longitudinal width expansion, leads to extremely non-uniform flow characteristics and will not be discussed, therefore.

Intuitively, it is guessed that under otherwise fixed flow conditions, the lateral outflow intensity slightly grows with decreasing contraction angle $\theta \leq 0$. This is easily realized by noting that the channel axis is deviated by the angle $\theta/2$ from the lateral weir plane. The total lateral outflow angle thus is $\phi_{\text{tot}} = \phi - 1/2 \theta \geq \phi$.

A corresponding effect on the lateral outflow angle is obtained for bottom slopes $S_0 < 0$ when compared to nearly horizontal channels. If subscript "tot" is dropped, Eq. 14 modifies to

$$\sin \left\{ \phi + \frac{1}{2} (\theta + S_0) \right\} = \left[ \frac{y - W}{3 - 2y - W} \right]^{1/2} = \Phi \quad \ldots \ldots \ldots \ldots (15)$$

Let $\lambda = (\theta + S_0)/2 < 1$ then, from trigonometrical arguments, $\sin (\phi + \lambda) = \sin \phi \cos \lambda + \cos \phi \sin \lambda = \sin \phi \cdot (1 - 1/2 \lambda^2) + \lambda \cdot \cos \phi = \Phi$. This may equally be expressed as quadratic equation in $\sin \phi$; after dropping terms in $\lambda^2$ the relevant solution reads

$$\sin \phi = \left[ \frac{y - W}{3 - 2y - W} \right]^{1/2} \cdot \left\{ 1 - \lambda \left[ \frac{3(1 - y)}{y - W} \right]^{1/2} \right\} \quad \ldots \ldots \ldots \ldots (16)$$

Since $|\lambda| << 1$, the second term of Eq. 16 is only corrective to the first term. For the usual configuration of a prismatic, nearly horizontal side weir, the lateral outflow angle is given by Eq. 14.

**Lateral Outflow Intensity.—**Upon combination of Eqs. 2 and 7, thereby accounting for $\omega$ as given previously, the lateral outflow intensity becomes (3)

$$Q' = \frac{3}{5} n c \sqrt{gH^3(y - W)^{3/2}} \left\{ 1 - \frac{W}{3 - 2y - W} \right\}^{1/2} \left\{ 1 - (\theta + S_0) \left[ \frac{3(1 - y)}{y - W} \right]^{1/2} \right\} \quad \ldots \ldots \ldots \ldots (17)$$
\( n^* \) is the number of outflow sides (one or two); \( c \) = the weir crest influence (\( c = 1 \) for a sharp-crested weir, \( c = 8/7 \) for zero weir height) according to Eqs. 4 and 5 for broad-crested and round-crested weirs. \( \theta \) and \( S_0 \) are the tangent of the width contraction angle and the bottom slope, respectively. It should be borne in mind that \( H \to h \) in the definition of \( \zeta_b, \zeta_r \) for side weir flow.

The corrective term in Eq. 17 equals twice the term as given for the outflow angle, Eq. 15. A rigorous approach indicates that this term should also be added to \( \omega_u \) in Eq. (9); the sum of the two thus corresponds to what the final result is (3).

**DISCUSSION OF RESULTS**

Eq. 17 replaces the usual weir formula Eq. 1 for the prediction of the side weir discharge characteristics. It should be noted that both of the formulations allow the one-dimensional approach. Once the side weir geometry \( (n^*, c, \theta, S_0, W \text{ as function of } x) \) is known and the local flow depth \( h \) and energy head \( H \) are computed both lateral outflow angle \( \phi \) and lateral outflow intensity \( Q' \) are easily obtained.

The following discussion refers to the usual case of a nearly horizontal, prismatic side weir, \( (\theta + S_0) \approx 0 \). Compared to the conventional approach \( Q' = -q \) according to Eq. 1, the new formulation yields

\[
\omega = \left[ \frac{1 - W}{3 - 2y - W} \right]^{1/2} \quad \text{........................................ (18)}
\]

as corrective term. It is seen that \( \omega = 1 \) for \( y = 1 \) (plane flow conditions) but has the minimum for \( y = W \), \( \omega = 1/\sqrt{3} \). Consequently, the lateral outflow intensity of a side weir is always smaller than for the corresponding flow conditions in a plane weir. The parameter \( \omega \) depends significantly on the local flow condition (\( y \)) and on the relative weir height \( W = w/H \). For fixed \( F \), \( \omega \) increases with decreasing \( W \). For zero weir height, \( W = 0 \), \( \omega \) is solely dependent on the local Froude number \( F \). Since both \( W \) and \( y \) vary gradually along the side weir, also the variation of \( \omega \) is smooth.

Few attempts have been put forward to incorporate dynamical effects in the lateral outflow equation. Coulomb, et al. (1) investigate the lateral outflow angle for a broad-crested weir; their result is Eq. 14 by an approach different from the present. The effects of \( \theta \) and \( S_0 \) are thus ignored. However the lateral outflow intensity is given as

\[
Q' = -\left( \frac{2}{3} \right)^{3/2} \sqrt{gH^3(1 - W)}\sqrt{3y - 2 - W} \quad \text{.................. (19)}
\]

thus

\[
\bar{\omega} = \left( \frac{1 - W}{y - W} \right) \left[ \frac{3y - 2 - W}{y - W} \right]^{1/2} \quad \text{................................ (20)}
\]

from Eqs. 2, 4, 7 for \( \zeta_b = 7/9 \). \( \bar{\omega} \) is only defined if \( y \geq (2 + W)/3 \). In rectangular channels with \( y(F = 1) = 2/3 \), supercritical flow conditions cannot be investigated. For the sharp-crested side weir, no definite results have been obtained.
Subramanya and Awasthy (11) consider the special case of a rectangular, prismatic side weir of zero weir height, \( W = 0 \). The discharge coefficient in Eq. 1 is given as

\[
\hat{C}_d = 0.407 \left[ \frac{2(1 - F_0^2)}{(2 + F_0^2)} \right]^{1/2} \quad ; \quad F_0 < 0.8
\]

\[
\hat{C}_d = 0.24 \left[ 1 - \frac{2}{9} F_0 \right] ; \quad F_0 > 2 \tag{21}
\]

in which \( F_0 = \frac{V_0}{(gh)}^{1/2} \) is the Froude number of the approaching flow. Nadesamoorthy and Thomson (8), in a discussion of the previous investigation, obtain

\[
\hat{C}_d = 0.407 \left( \frac{2 + F_0^2}{[2(1 + 2F_0^2)]} \right)^{1/2} \tag{22}
\]

It should be noted that, for both of the formulations, \( \hat{C}_d \) depends exclusively on the approaching flow conditions (index "0").

In contrast, Eq. 17 accounts for the local variation of the Froude number. With \( H = h + \frac{Q^2}{2gA^2} = h(1 + F^2/2) \) in the rectangular, prismatic channel, thus \( 1/y = 1 + (1/2) F^2 \), and for \( W = 0 \), Eq. 17 may be expressed as

\[
Q' = \frac{3}{5} \cdot \frac{8}{7} \left[ \frac{2}{2 + F^2} \left( \frac{2}{2 + 3F^2} \right)^{1/2} \right] n^* \sqrt{gh} \tag{23}
\]

in which \((8/7)\) is due to Eq. 3 for \( \chi = 1 \). The respective discharge coefficient then is

\[
\hat{C}_d = \frac{-Q'}{n^* \sqrt{2gh}} = 0.485 \cdot \left[ \frac{(2 + F^2)}{(2 + 3F^2)} \right]^{1/2} \tag{24}
\]

Eqs. 22 and 24 can only be compared by letting \( F \to F_0 \) (no significant change of \( F \) along the lateral outflow length). It is seen that \( \hat{C}_d \) is 20% higher for \( F \to 0 \) according to Eq. 24. These differences increase for increasing \( F \); asymptotically, \( \hat{C}_d (F_0 \to \infty) = 0.407/2 \) according to Eq. 22 and \( \hat{C}_d (F_0 \to \infty) = 0.485/\sqrt{3} \) according to Eq. 24.

Consequently, neither of the existing approaches incorporates all of the effects influencing the lateral outflow. Although the recommended new lateral outflow formula 17 is unhandy, only such a formulation may include geometrical as well as hydraulic properties of side weir flow (5).

Experiments

Facilities and Instruments.—The preceding theoretical relations have been compared with observations collected in a rectangular channel of width \( b = 0.30 \) m. A detailed description of the test facilities is given in Refs. 3 and 5. It suffices here to mention that the inflow Froude number has been varied between 0.3 \( \leq F_0 \leq 2 \) and that weir heights have been chosen constant between 0 \( \leq w \leq 0.20 \) m. The tests to be described have a single lateral opening of length \( \Delta L = 1.00 \) m. The bottom slope has
been adjusted to $-0.5 \leq S_0 \leq 2\%$. Herein, only the observations regarding the lateral outflow will be discussed, since other descriptions (3,5) refer to the internal flow in the side weir.

The free surface has been recorded with a precision gauge; velocities are measured with a mini-propeller of diameter 8 mm. The flow direction has been obtained with a thin, vertical plate fixed at a vertical axis at the upstream side. The movement of the probe (resembling to a flag) is mechanically transmitted to the top of the fixation pole and thus allows rapid determination of the local angle $\phi$ with respect to the channel axis. Aspects of the experimental uncertainty are presented at length in Ref. 3 and summarized in Ref. 5.

Typical Results.—Fig. 4 shows the lateral outflow velocity profiles at the (sharp-crested) weir plane for run B and J. Also indicated in the plot are the surface velocities. Run B corresponds to subcritical flow conditions ($F_0 = 0.62$), while run J is characterized by $F_0 = 2.06$.

The velocity profiles at the surface are fully turbulent, and the flow direction relative to the channel axis increases from the wall opposite to
the lateral outflow towards the outflow plane. Owing to the combination of channel flow and weir flow, the lateral outflow direction $\phi$ increases with decreasing depth at the outflow side. This typical flow property is particularly well seen in run B. Also evident from run B is the surface separation zone at the downstream end of the side weir; the downstream discharge for run B is $Q_d = 0$.

Fig. 5 compares the observed average lateral outflow angles $\phi_{ex}$ at various positions $x$ with the theoretically determined angles $\phi_{th}$ according to Eq. 16 for runs A to L (3). Also indicated are the $\pm10^\circ$ limits, and it is noted that all predictions are within these limits. There seems to be no systematic deviation from the prediction except for the lateral outflow sections situated at either end of the lateral. Evidently, flow characteristics at the up- and downstream end of the side weir are not gradually varied; for example the weir geometry $w(x)$ has a discontinuity.
Fig. 6(a) compares the lateral outflow intensity $Q'$ according to the observations (3) and based upon Eq. 17. It is seen that, except at the up- and downstream bounding cross sections, the prediction is always within ±10%. $Q'_h$, according to Eq. 17 overestimates normally $Q'_e$ at the inflow section, while $Q'_h$ is slightly too low at the downstream end of the side weir. However, these local errors are compensated when considering the total lateral outflow, $\Delta Q = \int_{AL} Q' dx$, when integrated over the length $AL$ of the side weir, see Fig. 6(b). The error in $\Delta Q$ is normally within ±5%.

**APPLICATION**

The results presented may simply be applied to particular cases of side weir flow. Herein, the dividing flow configuration in a rectangular channel will be studied for pseudo-uniform flow conditions, e.g., flow in a channel of bottom slope $S_0 > 0$, by which the frictional slope $S_f$ and the energy loss slope $S_b$ due to lateral flow are compensated. Furthermore, no lateral submergence is accounted for. Assuming nearly hydrostatic pressure and uniform velocity distributions, the energy head $H$ then remains constant with respect to the channel bottom. In a rectangular channel of width $B = b + \Theta x$ in which $B(x = 0) = b$ and $\Theta < 0$ is the tangent of the contracting angle, Bernoulli's equation reads

$$H = h + \frac{Q^2}{2gB^2h^2} \quad \text{.......................... (25)}$$

Differentiation with respect to the longitudinal coordinate $x$ and eliminating discharge $Q$ with Eq. 25 then yields

$$B(3h - 2H)h' = 2\Theta h(H - h) - \sqrt{\frac{2(H - h)}{g}} \cdot Q' \quad \text{.......................... (26)}$$

Let $m = n^*c$ be a factor by which the number of lateral openings and the weir shape (c = 8/7 for the case studied) are accounted for. Then, with

$$X = \frac{mx}{b}; \quad y = h \frac{H}{m}; \quad \Theta = \frac{\Theta}{m} \quad \text{.......................... (27)}$$

the non-dimensional equation for the free surface profile is

$$(1 + \Theta X)(3y - 2) = 2\Theta y(1 - y) + \frac{24}{35} \left( \frac{2y^3(1 - y)}{3 - 2y} \right)^{1/2}$$

$$\left\{ 1 - \Theta \left( \frac{3(1 - y)}{y} \right) \right\}^{1/2} \quad \text{.......................... (28)}$$

in which the prime denotes differentiation with respect to $X$. The control points, $X = 0$, correspond to $X(y = 1) = 0$ for $F < 1$ and $X(y = 2/3) = 0$ for $F > 1$ (5). Fig. 7 shows the free surface profile $y(X)$ obtained by a standard numerical integration procedure for $-0.4 \leq \Theta \leq 0$. Also plotted are normalized observations collected in a prismatic channel ($\Theta = 0$). It is seen that the maximum deviations from the prediction are always be-
low 5%. They are more significant at the inflow side weir zone and must be attributed to the neglect of the streamline curvature effects (4).

For run J, the experimentally observed average outflow angles are $\phi_{ex} = 23, 21, 21, 20, 19, 18$ degrees at intervals $\Delta x = 0.20$ m, compared to $\phi_{th} = 22, 20, 19, 17, 16, 15$ degrees according to Eq. 16. The proposed lateral outflow formulae thus reflect the effective flow behavior for both, sub- and supercritical flow conditions.

Evaluations comparable to Fig. 7 for weir heights $0 < W < 1$ are discussed in Eqs. 3 and 5. It is concluded that the proposed one-dimensional approach allows accurate modeling of the actual flow behavior. Excluding hydraulic jump formation, the average surface profile and the lateral outflow are predicted with ±5%.

CONCLUSIONS

The present investigation analyzes the lateral outflow mechanism of side weirs arbitrarily shaped by including the effects of flow depth, velocity of approach, lateral outflow angle and side weir channel shape. It is found that the usual weir formula for plane flow conditions is a special case for $F \to 0$; however, for $F > 0$, the aforementioned effects significantly influence the lateral outflow. The proposed Eqs. 16 and 17
for the average cross-sectional outflow angle $\phi$ and the lateral outflow intensity $Q'$ have been compared to observations, and a fair agreement is noted. The method of computation is illustrated by application of the formulae to bifurcations (side weir of zero height). The maximum deviations of the free surface profile are less than 5% with respect to the local energy head.

APPENDIX I.—REFERENCES


APPENDIX II.—NOTATION

The following symbols are used in this paper:

\[
\begin{align*}
A &= \text{cross-sectional area;} \\
B &= \text{channel width;} \\
b &= \text{channel width at control point;} \\
C_d &= \text{discharge coefficient;} \\
C_{d0} &= \text{basic discharge coefficient;} \\
c &= \text{weir shape factor;} \\
F &= \text{Froude number;} \\
g &= \text{gravitational acceleration;} \\
H &= \text{energy head;} \\
h &= \text{flow depth;} \\
L &= \text{length of broad-crested weir.}
\end{align*}
\]
$\Delta L$ = lateral outflow length;
$m$ = side weir factor;
$n^*$ = number of laterals;
$Q$ = discharge;
$Q'$ = discharge intensity;
$q$ = discharge per unit width;
$r$ = weir crest radius;
$S_f$ = friction slope;
$S_0$ = bottom slope;
$U$ = lateral outflow velocity;
$V$ = channel velocity;
$W$ = relative weir height;
$w$ = weir height;
$X$ = relative longitudinal coordinate;
$x$ = longitudinal coordinate;
$y$ = relative flow depth;
$\zeta_b$ = relative weir length;
$\zeta_r$ = relative weir radius;
$\Theta$ = relative convergence angle;
$\theta$ = channel convergence angle;
$\lambda$ = channel geometry characteristics;
$\rho$ = pressure coefficient;
$\Phi$ = modified lateral outflow angle;
$\phi$ = lateral outflow angle;
$\chi$ = relative weir height; and
$\omega$ = lateral outflow factor.