It is interesting that the author's $\omega_b$ can be written in terms of $F_w$. Apparently, $c$ accounts for the other parameters in Eq. 31. In analyzing their experimental results for discharge coefficients, some investigators have not considered all of the variables in Eq. 33.

Some additional references on side weirs are by Mostafa and Chu (1974), Ramamurthy and Carballada (1980), and Ramamurthy et al. (1978), who also considered angled side weirs ($\theta \neq 0$), as did Jain and Fischer (1982).

**ACKNOWLEDGMENTS**

The writer's work on side weirs was supported by the Harris County (Texas) Flood Control District. Jack E. Davis and Kevin A. Tynes, graduate research assistants on this project, contributed to the discussion. Appreciation is expressed to the author for an opportunity to discuss side weir flows with him.

**APPENDIX. REFERENCES**


**Closure by Willi H. Hager**

The author would like to thank Holley for his interest in the paper.

Regarding question 1, it has been stated that the "effect of flow depth" (page 484) is a transformation of $q(H)$ to $Q'(h)$. Therefore, the parameter $\omega_h$ obtains

$$\omega_h = \left( \frac{F_w^2 + 2}{3F_w^2 + 2} \right)^{1/2}$$

(32)

It is seen that $\omega_h$ reduces the lateral outflow considerably; for plane weir flow, $\omega_h = 1$. The combined effect of the velocity of approach and the flow

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depth indicates that \( \omega_u \cdot \omega_h = (h - w)/(H - w) \leq 1 \).

As concerns question 2, no limits of application were imposed on Eq. 9, if the local energy head \( H = H(x) \) is considered. The author disagrees with the writer in so far as the effect of the velocity (of approach) increases the lateral outflow \( Q' \), but that the effect of \( \omega_h \) is stronger than \( \omega_{\nu u} \). The result is a net decrease of \( Q' \) when compared to \( q \) (plane flow condition).

The application of Eq. 14 to broad-crested weirs (question 3) was not verified experimentally. However, Coulomb et al. (1967) derived an expression for the broad-crested weir that is identical to Eq. 14. It seems, therefore, that the relation in question applies (at least approximately) to flow over broad-crested side weirs.

As shown by Hager (1986), the residual pressure coefficient becomes \( \rho = 2/3 \) for weir flow when accounting for the curved streamline pattern, whereas \( \rho = 5/6 \) if the pressure distribution would be hydrostatic. \( \rho = 5/6 \) thus accounts for the depth of flow at the crest section, whereas \( \rho = 2/3 \) refers to the pressure head [see also Fig. 2(d)].

The effect of negative bottom slope \( S_a < 0 \) and the contraction angle \( \theta < 0 \) on \( Q' \) are addressed in question 5. Both effects are similar in that the lateral outflow angle \( \phi \) is increased by \( -S_a/2 \), and \( -\theta/2 \). As stated by the discusser, "there is a component of \( V \) that is now normal to the weir crest, thereby allowing the longitudinal momentum . . . to contribute to the flow over the weir." It should be noted that both \( S_a \) and \( \theta \) have only a second-order effect on \( \phi \) and \( Q' \). The role of the factor

\[
H'_s = -\left( \beta - \frac{U \cdot \cos \phi}{V} \right) \frac{QQ'}{gA^2}
\]

is well understood as regards its influence on the head-loss due to the lateral outflow (Hager and Volkart 1986). The average gradient due to lateral flow \( H'_s \) may be correlated to the head-loss coefficient \( \xi \) as

\[
\frac{\Delta H_s}{\Delta L} = \xi \frac{V_u^2}{2g\Delta L}
\]

where \( \Delta L \) is the lateral outflow length, and \( V_u = \) side weir approaching velocity. Fig. 8 shows \( \xi \) as a function of the lateral outflow \( \Delta Q \) relative to the inflow \( Q_u \) (Hager 1981). It is seen that

\[
\xi = \frac{4}{5} \frac{\Delta Q}{Q_u} \left( \frac{\Delta Q}{Q_u} - \frac{1}{2} \right)
\]

as given by Hager and Volkart (1986) describes reasonably well the data.

The disadvantage in formulating the head loss due to lateral flow by Eqs. 35 and 36 is that only the average head-loss gradient \( \Delta H_s/\Delta L \) may be computed, whereas \( H'_s \) refers to the local head loss. To overcome this deficiency, several trials were made to correlate \( \Lambda = \beta - U \cdot \cos \phi/V \) with another significant parameter. Finally, the plot as shown in Fig. 9 was found to describe reasonably well the sought relation. Herein, the unknown is correlated to

\[
R = \frac{\bar{q}}{1 + \bar{q}}, \quad \text{where} \quad \bar{q} = \frac{|Q'B|}{Q}
\]
FIG. 8. Head Loss Coefficient $\xi$, as Function of Relative Lateral Outflow, $\Delta Q/Q_u$:
(—) Eq. 36; (◇) Run A to $U$; (□) Run I to XV; and (■) Run 1 to 21 (Hager 1981)

Clearly, $\bar{q}$ describes the lateral outflow per unit channel length relative to the local discharge per unit channel width; $\bar{q}$ varies between zero and infinity, such that $0 \leq R \leq 1$. $R = 1$ corresponds to lateral outflow at the downstream extremity of a dead-end channel.

The linear approximations

FIG. 9. $\beta - U \cdot \cos \phi / V$ as Function of $R$; Notation as in Fig. 5; (—) Eq. 38
$\Lambda = 0.2$, \hspace{1em} $0 < R < 0.083$, \hspace{1em} $\Lambda = 0.4 \left(1 - 6R\right)$,

$0.083 < R < 0.5$ \hspace{1em} (38)

may be used. The data refers to runs $A$ through $K$ and include both prismatic and nonprismatic channel geometries. In all runs the parameters $\beta$, $U$, and $\phi$ have been obtained experimentally by 3-D velocity measurements. The range of local Froude numbers covers $0 < F < 3.1$. Additional data (not included in Fig. 9) indicated a fair agreement with Eq. 38, particularly for small $R < 0.1$. The domain $0.5 < R < 1$ is not covered by Eq. 38, since it is difficult to observe such highly spatial flow phenomena, as reported by Hager and Volkart (1986). As an example, run $C$ as shown in Fig. 7(a) in the previously mentioned paper has $R$ values larger than 0.5 for $3.8 \text{ m} < x < 4.0 \text{ m}$. Run $L$ was excluded in Fig. 9 because of the adverse sloping bottom geometry ($S_0 = -0.2$). How far both $S_0$ and $\theta$ influence $\beta - U \cdot \cos \phi/V$ may not be definitely settled, since more experimental data are needed.

Eq. (38) is useful for the prediction the free surface profile $h(x)$, and the local discharge distribution $Q(x)$ in side-weir flow, since quantities that may only be obtained by a 3-D approach are approximated by quantities that appear in the one-dimensional approach (Hager and Volkart 1986).

Let us now discuss some interesting features of Eq. 38. Consider small $\bar{q}$-values, such that $R \rightarrow \bar{q}$ and

$\Lambda = 0.2$ \hspace{1em} (39)

Therefore, the head-loss gradient due to lateral flow becomes

$H' = -0.2 \frac{QQ'}{gA^2}$ \hspace{1em} (40)

noting that $Q' < 0$ indicates that $H'_1 = 0$. Side-weir flow of which the local discharge per unit width $Q/B$ is much larger than the lateral discharge intensity $Q'$ thus generates an increase of the energy head in the channel flow. This increase of energy is typical for certain junction flows, as is also reflected by negative head-loss coefficients. Clearly, this increase of energy in the channel flow is associated with a corresponding energy decrease in the lateral flow. The question if the energy line $H(x)$ is increasing or decreasing depends on the sum $H' = -S_f + H'_1$. Usually, the friction slope $S_f$ will be larger than $H'_1$ and a net decrease of $H(x)$ results. However, there may be cases with subcritical flow where $H' > 0$ can be observed, at least at the side-weir upstream zone. Recently, such phenomena have been verified for flows in distribution conduits (Hager 1986).

To close the discussion, the system of equations for side-weir flow becomes

$H = z + h + \frac{Q^2}{2gA^2}$ \hspace{1em} (41)

$H' = -S_f - \Lambda \frac{QQ'}{gA^2}$ \hspace{1em} (42)

$Q' = -\frac{3}{5} n^*c \sqrt{gH^3} (y - W)^{3/2} \left[ \frac{1 - W}{3 - 2y - W} \right]^{1/2}$. [Continued]
\[
\left\{ 1 - (\theta + S_o) \left[ \frac{3(1 - y)}{y - W} \right]^{1/2} \right\}^{3d - v) /2}
\]

(43)
in which \(S_o = -dz/dx\) and \(S_f\) is computed either with the Colebrook-White equation or any appropriate simplification. Imposing two boundary conditions \(h(x = x_0) = h_0\), and \(Q(x = x_0) = Q_0\) then allows to solve for the unknowns \(h(x)\) and \(Q(x)\) by a numerical approach.

**APPENDIX. REFERENCE**


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**VELOCITY AND DISCHARGE IN COMPOUND CHANNELS**

Discussion by Y. Lam Lau and B. G. Krishnappan

The writers are rather intrigued by the author's finding that the average velocity in the flood plain exceeded that for the main channel for certain ratios between the flood plain depth and the main channel depth. This appears to be in contrast to the data of Knight and Demetriou (4), who made measurements in channels of approximately the same dimensions as those of the author. To the writers' knowledge, this phenomenon, indicating the transfer of momentum from the flood plain to the main channel, has not been reported by other investigators.

The writers used a three-dimensional turbulence model to compute the shear stress and flow distributions in rectangular channels with flood plains (18). The results compared favorably with the data of Myers and Elsawy (19), Knight and Demetriou (4), and Wormleaton et al. (15). Using this model, the velocity distributions for the three cross sections used in the author's experiments were calculated. The average velocities for the different sections were then obtained from which the ratios of flood plain and main channel velocities to the cross section velocity were calculated. These ratios are plotted against the relative depth, \(y_r\), in Fig. 11. These results do not exhibit the cross over of velocity distributions that were shown in Fig. 5. Also plotted are the curve of \(v_f/V\), which is calculated from Eq. 9, the empirical equation obtained by Knight and Demetriou and the curve of \(v_f/V\), which is given by the following equation:

\[
\frac{v_f}{v} = 1 + \frac{(1 - \frac{v_c}{V})}{(\alpha - 1)\beta}
\]

(15)

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