Flow Patterns in a Four-Branch Junction with Supercritical Flow

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Abstract: This paper describes the flow structures that occur in a 90° junction of four open channels with supercritical flow in two orthogonal inlet channels. An experimental facility was constructed to permit the measurement of flow rates, water depths, and the positions of hydraulic jumps in the channels. The various flow patterns which appear depend on the characteristics of the incoming flows and can be classified into three main types, depending on the location and shape of the hydraulic jumps that develop. These jumps can either be normal to the flow and located in the upstream channels, or can be oblique and confined within the junction. The explanation for the existence of various flow patterns is derived from previous studies of the rapid deflection of supercritical flow. A detailed description of each flow regime is provided, with information on the surface elevations, the behavior of the hydraulic jumps, the deflections of the incoming flows, and the formation and characteristics of recirculation zones.

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Introduction

When severe flooding occurs in an urban area, most of the flow is carried as surface water in the streets (Haider et al. 2003; Inoue et al. 2000; Ishigaki et al. 2004), and in some areas, the slope of the streets can be steep enough to create supercritical flow (Mignot et al. 2006). Intersections play an important role in distributing the flow and in defining upstream and downstream boundary conditions for the flow in individual streets, but very little information is currently available on the behavior of supercritical flows in channel junctions. The flow in an intersection is typically highly perturbed, and strongly three dimensional, so it is difficult to simulate numerically. But the high water marks left after the flood are often used to calibrate numerical models (Mignot et al. 2006; Smith et al. 2005), so a detailed understanding of flow in an intersection can be important in interpreting data from flood records.

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Most of the published studies of free surface flow in 90° channel junctions concern the intersection of three channels, in the form of a T, with results for both subcritical and supercritical conditions. Descriptions of flow patterns in completely subcritical junction flow can be found in Weber et al. (2001) for confluences, and Neary et al. (1999) for separations. The main flow structures that characterize these configurations are zones of recirculation, some induced secondary circulations, shear separation planes, and flow contractions. Although we do not consider the influence of channel geometry in this study, it should be pointed out that this aspect has been investigated in the context of geomorphologic studies. Recent reviews can be found in Rhodes and Sukhodolov (2001), who emphasized the role of turbulent mixing, in Biron et al. (2004) who discussed the influence of differences in bed level, and in Kenworthy and Rhodes (1995) who investigated sediment patterns.

One of the first studies of confluent supercritical flow in a three-channel junction was reported by Bowers (1950) who showed that the formation of hydraulic jumps in the inlet channels of the junction depends on the junction geometry and the upstream flow rates. He found that the position of the jump is dependent upon the flow rates in the incoming channels and when a jump does not form, waves are created at the junction. He also observed that decreasing the discharge in the main inlet channel causes the jump in that channel to move upstream and causes the jump in the side channel to move downstream. Behlke and Pritchett (1966) studied the characteristics of diagonal waves that form in the junction, and observed that these waves have their origin at the upstream corner of the junction. Rice (1985) performed an experimental study of the influence of geometrical parameters and flow characteristics on the location of the hydraulic jump. Hager (1989) analyzed supercritical flow in a three-channel junction using an approach similar to that of Behlke and Pritchett (1966) in which it is assumed that both incoming flows are suddenly deflected when they reach the oblique wave fronts in the junction. This enabled him to provide a precise description of the flow in the junction. Schwalt and Hager (1995) described the main fea-
tures of the waves forming at the junction of two supercritical flows with a low junction angle.

Supercritical flow in a junction formed by the orthogonal intersection of four channels has been studied by Nania et al. (2004), and, as far as we are aware, no one else. They studied a configuration with two inlet channels and two outlet channels, and found that the different flow patterns that were generated could be classified into two types—Type I, for which two jumps form in the inlet channels, and Type II for which one jump forms in one of the inlet channels and another forms in the intersection. They did not provide detailed measurements of the flow patterns, nor the conditions necessary for the formation of the two flow types.

The present study extends the work of Nania et al. (2004) in three ways. We identify and define a third flow type, we provide a detailed description of each flow pattern observed experimentally, and we explain the physical mechanisms responsible for the different flow regimes.

There are three major sections to this paper; in the first section, we describe the experimental installation, operating conditions, and measurement techniques. In the second section, we provide a brief description of the different flow types, with particular emphasis on the influence of the different hydraulic jumps that are observed. Finally, we provide a detailed description of each major flow type, together with some details of intermediate configurations and additional flow features such as stagnation zones and wave reflections.

**Experimental Setup and Measuring Techniques**

The experiments were performed in the channel intersection facility at the Laboratoire de Mécanique des Fluides et d’Acoustique at the Université de Lyon (Figs. 1 and 2). The facility consists of four glass channels, 0.3 m wide and 2 m long, with slopes that can be varied independently between −5 and +5%. The four channels intersect at 90°, and the base of the intersection remains horizontal, irrespective of the slopes of the channels. In these experiments, two of the channels (at right angles) provide the inlet flow, and the other two act as outlet channels. Each inlet channel is fed from a large storage tank at the upstream end, where a honeycomb serves to stabilize and straighten the inlet flow. In most of the experiments the flow
depth in the inlet channel was imposed using a vertical sluice gate located just downstream from the channel inlet.

The main parameters measured in these experiments were the discharges in the four channels, the surface profiles in the region of the intersection, and the locations of the hydraulic jumps. The flow rates in the channels were measured using four identical electromagnetic flowmeters (Promag 50 from Endress Hauser; accuracy of ±0.02 L/s), connected to a personal computer. Where the flow was uniform, the water depth was measured using a point gauge with an accuracy of ±0.15 mm. This device could not be used to measure the depth in regions where there were strong eddies or hydraulic jumps because the water surface was highly disturbed and unsteady, so in those zones we have used a movable wave probe (wave monitor from Churchill Inst.). For this probe, the gap between the two vertical parallel wires is 10 m and the probe position in the horizontal plane was measured to an accuracy of ±1 mm. The accuracy of the water level measurements, including the effects of wetting, wake formation, and the vertical positioning of the probe, was estimated as ±0.5 mm. This estimation was made from comparisons with point gauge measurements in supercritical, fully developed flows at various Froude numbers. To ensure that the experimental conditions were stationary, the flow rates in the four channels were monitored continuously and the measurements only began when the discharges were constant in time and the inlet and outlet mass flow rates balanced.

The experiments described in this paper were performed for four different slope (S) configurations: $S_x=S_y=1\%$; $S_x=S_y=3\%$; $S_x=5\%$, $S_y=1\%$, and $S_x=S_y=5\%$. The slopes of the inlet and outlet channels in any given direction were identical. The discharges in the two inlet channels were controlled by valves in the pumping circuit, and could be set independently to values in the range of 0–10 L/s. In the standard configuration described here, the inlet conditions were limited to Froude numbers less than 4.5, where the Froude numbers for the flow in the $x$ and $y$ inlet channels are defined as

$$F_{xi} = \frac{Q_{xi}/bh_{xi}}{\sqrt{gh_{xi}}}, \quad F_{yi} = \frac{Q_{yi}/bh_{yi}}{\sqrt{gh_{yi}}}$$

(1)

To obtain higher Froude numbers, the configuration was modified slightly: the four channels were set horizontally and the inlet sluice gates were placed about 0.45 m upstream from the entry to the junction [Fig. 1(c)]. In this configuration, it was possible to obtain inlet Froude numbers up to 5.5. In total, more than 200 experimental configurations were studied.

The depth in the inlet channel was set to the normal depth $h_n$ for the chosen, imposed flow rate, and bed slope, using the vertical sluice gate at the inlet. This normal depth was computed using values of Manning’s $n$ estimated from the Darcy–Weisbach friction factor, obtained from the Colebrook–White equation, for flow that is turbulent and hydraulically smooth—the Reynolds numbers ranged from 6,500 to 15,000, and the channel floor and walls were made of glass. This gave values for Manning’s $n$ in the range $0.0087–0.0074$ s m$^{-1/3}$, and normal depths that were always less than 30 m, so the ratio of channel length $L$ to depth $h$ was in the range $50 < L/h < 100$, which is sufficient to ensure fully developed flow in the channel (Ranga Raju et al. 2000), in the absence of any disturbance caused by the channel intersection. The Weber number in these experiments always exceeded 170 so surface tension effects can be neglected.

The experimental configuration used here is quite similar to that used by Nania et al. (2004) but with smaller dimensions—the channels here are 0.3 m wide, whereas the experiments of Nania et al. (2004) were conducted in channels 1.5 m wide. However, in our experimental setup, the inlet channels are long enough to ensure that uniform flow develops; this was not the case for the experiments of Nania et al. (2004).

The aspect ratio (width/depth) lies in the range of 10–35. This is coherent with values of the aspect ratio that occur in real urban flooding; in the floods in Nîmes in 2002, for example, the average water depth in the streets was on the order of 0.3 m, and the street width on the order of 10 m, giving a typical aspect ratio on the order of 30–35 (Mignot et al. 2006). The channel widths are small compared with those of real streets so scale effects might modify the similarity between the flow behavior observed experimentally and that observed at real scale. Nevertheless, the results are in good qualitative agreement with the observations of Nania et al. (2004) obtained for aspect ratios in the range 10.5–115.

**Different Flow Patterns**

**Three Flow Types**

Due to the steep slopes used in these experiments, the upstream uniform flows are supercritical and would remain supercritical in the absence of any singularity. However, when the inlet flows reach the intersection they collide and are rapidly deflected, creating two hydraulic jumps. Downstream of the junction, contraction and recirculation zones appear along with standing waves, which reflect from the sidewalls. Further downstream, the flow returns to a one-dimensional state.

The experiments show that the flow patterns depend on the discharges and the channel slopes. Three major types of flow can be identified (Fig. 3); two of these (Types I and II) have already been reported in Nania et al. (2004) and we have identified a third type, which we have named Type III.

Type I is a flow pattern for which a normal hydraulic jump occurs in each of the inlet channels (Fig. 4), and both inlet supercritical flows therefore become subcritical before reaching the junction. The flow within the junction is subcritical, and returns to a supercritical regime in the outlet channels.
Type II is a flow pattern for which a normal hydraulic jump occurs in the inlet channel carrying the weaker flow and an oblique jump occurs within the junction (Fig. 5). At the entry to the junction the main flow is supercritical and the weaker flow is subcritical. Downstream of the oblique jump, the flow conditions may be either supercritical or subcritical, depending on both the upstream Froude number and the angle of deflection induced by the weaker flow.

Type III is a flow pattern for which two oblique jumps appear within the junction (Fig. 6). The inlet flows are supercritical at the entry to the junction and can become subcritical or remain supercritical downstream of the jumps. In the outlet channels, the flows return to a supercritical regime.

Deflection of Supercritical Flow

Ippen (1951) studied the behavior of a supercritical channel flow that is suddenly deflected through an angle $\theta$. He found that this leads to the formation of a standing wave—an “oblique hydraulic
jump”—at an angle $\beta$ to the direction of the incident flow, where $\beta > \theta$ (Fig. 7). He showed that the two angles are related through

$$\tan \theta = \frac{(\sqrt{1 + 8F^2 \sin^2 \beta} - 3) \tan \beta}{2 \tan^2 \beta + \sqrt{1 + 8F^2 \sin^2 \beta} - 1} \tag{2}$$

where $F$ = Froude number of the incident flow. Now for any given $F$ this equation can only be solved for the jump angle $\beta$ provided that the deflection angle $\theta$ is less than some limiting value $\theta_{\text{max}}$, which is a function of $F$, and increases with $F$. The relationship between $\theta_{\text{max}}$ and $F$ has been plotted in Fig. 8. An oblique jump will only form if $\theta < \theta_{\text{max}}$; if the imposed deflection $\theta$ is larger than $\theta_{\text{max}}$, the hydraulic jump detaches from the source of the deflection, and the jump resembles a bow wave composed of a small normal jump, a circular jump, and finally an oblique jump. Downstream, where the flow returns to a subcritical regime, the deflection of the flow is imposed by the curvature of the streamlines.

In the junction, the two supercritical inlet flows collide and deflect each other. Behlke and Pritchett (1966) suggest that the
The specific momentum is a better parameter to use to characterize the flow than the flow power because it is conserved across the hydraulic jump. Also, since it is analogous to a force, it seems more directly relevant to the idea suggested by Behlke and Pritchett (1966) that the influence of the incoming flow could be modeled by an equivalent, angled side wall.

In the following paragraphs we consider two asymptotic cases, based on the relative momentum of the flow in the two inlet channels.

Case 1 corresponds to quasi-symmetrical configurations in which the slopes are equal in both directions and the discharges in the two inlet channels \( Q_x \) and \( Q_y \) are similar. It follows that the specific momenta of the flows in the two inlet channels are also similar: \( M_x = M_y \) and both deflections \( \theta \) and \( \theta' \) are close to 45°.

Case 2 corresponds to those configurations for which the specific momentum of the flow in the main channel is much greater than the specific momentum of the flow in the side channel; this condition may be (but is not necessarily) accompanied by a difference in the channel slopes. For this configuration, the main flow deflection \( \theta \) becomes very small and the deflection of the minor flow (\( \theta' \)) tends to 90°.

**Case 1 (Quasi-Symmetrical Configuration)**

In quasi-symmetrical conditions, the deflection angles for the two incoming flows must be close to 45°, irrespective of channel slope and the specific momentum of the flow. But the maximum deflection angle that can be produced by an oblique jump depends on the Froude number of the incoming flows. For relatively small channel slopes (such as the configuration \( S_x = S_y = 1\% \)), the inlet Froude numbers are relatively small (typically of the order of two) so the maximum possible flow deflection given by Eq. (2), as plotted in Fig. 8, is also small (\( \theta_{\text{max}} \approx 20° \)). This is much smaller than the actual deflections of the incoming flows (which are close to 45°), so the only way that such deflections can be achieved is by a transition from the supercritical to subcritical regime in the two upstream branches, this transition being effected by two normal hydraulic jumps. This is a Type I flow pattern, and is illustrated in Fig. 3(a).

**Links between Deflection of Incoming Flows and Flow Patterns**

First, it should be noted that in these experiments, for any fixed channel slope, the upstream Froude number \( F \) varies little with the inlet discharge. For instance, for the slope configuration \( S_x = S_y = 5\% \), and a discharge \( Q = 1 \, \text{L/s} \) with uniform flow in the channel, \( F = 3.81 \) whereas for a discharge \( Q = 5 \, \text{L/s} \), \( F = 4.3 \). So providing the flow regime remains uniform, the Froude number will change little and the maximum deflection angle \( \theta_{\text{max}} \) given by Eq. (2) will also remain almost constant. For example, for the two cases just cited, a flow rate of 1 L/s corresponds to \( \theta_{\text{max}} = 39° \) while the flow rate of 5 L/s corresponds to \( \theta_{\text{max}} = 42° \). The occurrence of the different flow patterns is determined by the ratio between \( \theta_{\text{max}} \) (which depends on the Froude number, as shown in Eq. (2)) and the deflection angle imposed by the other inflow. The deflection angle \( \theta \) depends on the ratio of the specific momenta of the two incoming flows, where the specific momentum \( M \) is given by

$$ M = \frac{Q^2}{bh} + \frac{gbh^2}{2} $$

and \( \rho = \text{fluid density} \). Then the specific momentum ratio in the \( x \) direction \( M_{rx} \) is defined as

$$ M_{rx} = \frac{M_x}{M_x + M_y} $$

So the flow patterns are determined by a combination of \( F \) (or, equivalently, the channel slope \( S \)) and the specific momentum ratio \( M_{rx} \). Nania et al. (2004) used the flow power \( W \) rather than the specific momentum to characterize the flow, where \( W \) is defined as

$$ W = \rho g Q \left( z + h + \frac{v^2}{2g} \right) $$

and \( v = \sqrt{Q/bh} \) = cross-sectional average velocity. The corresponding flow power ratio \( W_{rx} \) is defined as

$$ W_{rx} = \frac{W_x}{W_x + W_y} $$

The specific momentum is a better parameter to use to characterize the flow than the flow power because it is conserved across the hydraulic jump. Also, since it is analogous to a force, it seems more directly relevant to the idea suggested by Behlke and Pritchett (1966) that the influence of the incoming flow could be modeled by an equivalent, angled side wall.
For large inlet channel slopes, the upstream Froude numbers are large (greater than five) so \( \theta_{\text{max}} \) given by Eq. (2) is correspondingly large (and can exceed 45°). Under such conditions it is possible to deflect the two incoming flows by means of oblique jumps, so we find two oblique jumps within the junction, as illustrated in Fig. 3(c). Since this condition has not previously been observed we have named it a Type III flow pattern.

**Case 2 (Asymmetrical Configurations)**

In the asymmetrical configuration, the momentum of the main flow is significantly greater than the momentum of the side flow; thus, the range of specific momentum ratio \( M_x/M_t \) for which various flow types were observed experimentally is presented in Table 1. The range of \( M_x/M_t \) corresponding to each regime is approximate, because it is based on a classification of the conditions used in these experiments, rather than on a program of tests designed to define the boundaries between regimes. Nevertheless, the range of \( M_x/M_t \) corresponding to each regime clearly depends on the channel slopes and hence on the Froude number. It should also be noted that the transition between two types due to an increase (or decrease) of \( M_x/M_t \) is not abrupt; intermediate flow types exist when at least one detached hydraulic jump (similar in form to a bow wave) forms in the intersection. A description of the three different regimes and the regime transitions is provided in the following section.

### Table 1. Range of Specific Momentum Ratio \( M_x/M_t \) for Which Various Flow Types Were Observed Experimentally

<table>
<thead>
<tr>
<th>Slopes</th>
<th>Type I</th>
<th>Type II</th>
<th>Type III</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_x = S_y = 1% )</td>
<td>[0.25–0.75]</td>
<td>[0.25–0.75]</td>
<td>—</td>
</tr>
<tr>
<td>( S_x = 5% ), ( S_y = 1% )</td>
<td>[0.40–0.45]</td>
<td>[0.45–0.55]</td>
<td>—</td>
</tr>
<tr>
<td>( S_x = 5% ), ( S_y = 5% )</td>
<td>[0.45–0.55]</td>
<td>[0.45–0.55]</td>
<td>—</td>
</tr>
<tr>
<td>( S_x = S_y = 0 ) with ( F_x = 5.5 )</td>
<td>—</td>
<td>—</td>
<td>0.49</td>
</tr>
</tbody>
</table>

**Boundaries between Different Flow Types**

The different parameters (configuration, slopes, and momentum ratios) corresponding to these three flow regimes are presented in Table 1. The range of \( M_x/M_t \) corresponding to each regime is only approximate, because it is based on a classification of the conditions used in these experiments, rather than on a program of tests designed to define the boundaries between regimes. Nevertheless, the range of \( M_x/M_t \) corresponding to each regime clearly depends on the channel slopes and hence on the Froude number. It should also be noted that the transition between two types due to an increase (or decrease) of \( M_x/M_t \) is not abrupt; intermediate flow types exist when at least one detached hydraulic jump (similar in form to a bow wave) forms in the intersection. A description of the three different regimes and the regime transitions is provided in the following section.

### Three Regimes

#### Type I

For the Type I regime, both hydraulic jumps are located in the inlet channels. An important parameter characterizing the flow structure is the distance between the jumps and the junction. This can be of practical importance in assessing the possible impact of flooding in urban areas, because the jumps create a large increase in water depth and a sudden decrease in velocity. Nania et al. (2004) observed that an increase in the inflow power ratio in a street causes the corresponding jump to travel downstream (and vice versa), which is also the case in these experiments; however the data set additionally shows that for the Type I configuration with \( S_x = S_y = 1\% \), the distance between the normal jumps and the junction varies linearly with the specific momentum ratio \( M_x/M_t \).

**Fig. 10.** Distance between hydraulic jump and junction entry section: (■) hydraulic jump in \( x \) upstream branch; (○) hydraulic jump in \( y \) upstream branch. Type I configuration, with \( S_x = S_y = 1\% \).

(Fig. 10). As the slopes of the channels are equal, the data are approximately symmetrical relative to the line \( M_x/M_t = 0.5 \).

#### Type II

For Type II flow patterns, an oblique jump forms within the junction, across the major flow, and a normal jump forms in the inlet channel carrying the minor flow (see Fig. 11, for example). As the main flow crosses the intersection, it collapses into the outlet side channel, in a way that is similar to the flow created by the collapse of a dam (Hager and Yasuda 1997; Rivière and Perkins 2004) and a depression wave propagates from corner \( D \) (in Fig. 11) to the flow centerline. There are various possible interactions between the jump and the depression wave which suggests that the Type II regime can be divided into three subregimes. Subregime 1 corresponds to no interaction between the depression wave and the jump, subregime 2 corresponds to a weak interaction, and subregime 3 corresponds to a strong interaction.

#### Type II Subregime 1

Type II subregime 1 occurs when the deflection angle of the main flow is small; the supercritical jet entering the junction collapses across the line \( B-D \) in Fig. 11 and there is no interaction with the oblique jump. This flow corresponds to the geometrical condition \( \beta < 90° - \lambda \), where the angle \( \lambda \) (in degrees) is defined in Fig. 11.
The point $B$ is located at the intersection of the depression wave $(D-B)$ with the line defining the downstream exit section in the main channel. The position of the point $B$ (and hence the value of the angle $\lambda$) can be obtained from the results for critical flow in a three-branch intersection, for which the distance $B-C=b/F_{xi}$ where $b=$channel width and $F_{xi}=$Froude number for the main flow upstream of the junction (Rivière and Perkins 2004). The flow in this subregime is similar to that in the three-branch intersection because the depression wave is completely unaffected by the oblique jump; the lateral inflow $Q_{yi}$ and the deflection angle $\theta$ have no influence on the jet development, and the lateral outlet discharge $Q_{yo}$ will be the same as that for a three-branch separation in the supercritical regime.

Fig. 12 shows the surface profile in the intersection, measured with the wave probe. The depression wave corresponding to a sudden lateral water depth decrease is clearly visible, as are the normal jump and the oblique jump; the figure shows clearly that the oblique wave does not intersect the depression wave formed by the collapse of the main flow into the side channel. There is some eddying motion in the main channel downstream of the intersection, and the recirculation zone in the minor downstream flow channel is dry, so the depth measurements in these zones are less accurate.

**Type II Subregime 2**

As the deflection angle $\theta$ increases the angle $\beta$ of the oblique jump will also increase, and eventually the oblique jump will intersect with the depression wave somewhere within the intersection (Fig. 13). At this point the jet development is no longer perfectly free and some of the streamlines deflected towards the side channel by the developing jet are modified due to the interaction with the oblique jump in the vicinity of point $C$. This regime forms when $90^\circ-\lambda<\beta<45^\circ$. Fig. 14 shows the corresponding water surface profile measured in Type II subregime 2; the intersection between the depression wave and the oblique jump in the junction is clearly visible. Since the oblique jump angle $\beta$ depends partly on $Q_{yo}$, this parameter now has an influence on the jet development feature.

**Type II Subregime 3**

If the deflection angle $\theta$ increases still further, the oblique jump terminates in the minor channel downstream of the intersection (Fig. 15). In this case, only part of the discharge in the exit side channel ($Q_{yo}$) comes from the jet development, with the remainder coming from the main flow $Q_{xi}$ that is deflected while crossing the oblique jump. As the deflection angle increases, the fraction of $Q_{yo}$ provided by the deflection of the main flow also increases and the contribution from the jet development decreases. Correspondingly, the fraction of $Q_{xi}$, which continues into the main channel downstream of the intersection decreases. This regime forms when $45^\circ<\beta<\beta_{\text{max}}$, where $\beta_{\text{max}}=$angle of the oblique jump corresponding to the maximum deflection $\theta_{\text{max}}$, as given by Eq. (2). Fig. 16 shows the surface profile for the Type II subregime 3 configuration. We see that the depression wave is no longer the most important feature at the downstream side of the main flow. A comparison of the different Type II subregimes 1, 2, and 3 (Figs. 12, 14, and 16), shows that as the jump angle $\beta$ increases, the width of the oblique jump also increases while the distance...
between the normal jump in the inlet side branch and the entry to the intersection decreases. Furthermore, it should be noted that the water depth increases in the main channel near corner $C$ where the deflection wave impacts on the channel walls. The amount by which the water level rises near corner $C$ increases as the angle $\beta$ of the oblique jump increases. Thus the ratio between this maximum water depth and the normal depth in the main flow channel is highest for Type II subregime 3 (600% of the normal depth for the upstream flow) and lowest for Type II subregime 1 (400%). This is probably because the velocity of the flow deflected towards the corner $C$ is greater in Type II subregime 3 than it is for the jet development flow in Type II subregime 1.

Hydraulic Jump Behavior

For increasing $M_{rx}$ ($=M_x/M_t$) the deflection angle $\theta$ decreases and, from Eq. (2), the oblique jump angle $\beta$ also decreases. Correspondingly, the distance between the normal jump and the junction in the minor flow channel increases. Fig. 17 shows the evolution of the oblique jump angle $\beta$ with the specific momentum ratio $M_x/M_t$ for a Type II flow regime with the slope configuration $S_t=S_r=5\%$.

![Image](341x637 to 545x756)

**Fig. 17.** Oblique jump angle $\beta$ as function of upstream specific momentum ratio $M_x/M_t$ for the slope configuration $S_t=S_r=5\%$

We can see from Fig. 17 that the evolution of $\beta$ with $M_x/M_t$ is linear, and this might appear to conflict with the results reported by Nania et al. (2004), who found that $\beta$ varied linearly with the upstream power flow rate $W_x/W_f$ since, as a comparison of Eqs. (3) and (5) shows, it is not generally possible for a quantity to vary linearly with both the specific momentum ratio and the specific power ratio. However, the ratio of the specific momentum ratio $M_{rx}$ [Eq. (4)] to the fluid power ratio $W_{rx}$ [Eq. (6)] for a horizontal junction (i.e., $z=0$ for both inlet channels) can be written

$$\frac{M_{rx}}{W_{rx}} = \frac{1 + \left( \frac{F_y}{F_y} \right) \left( \frac{2 + F_x^2}{2 + F_y^2} \right)}{1 + \left( \frac{h_x}{h_y} \right)^2 \left( \frac{1 + 2F_x^2}{1 + 2F_y^2} \right)}$$

(7)

In these experiments the Froude numbers $F_y$ and $F_y$ did not vary much, and were not very different from each other, so this ratio always remained close to 1, and the variation of $\beta$ with $W_{rx}$ will be similar to the variation with $M_{rx}$.

For this configuration ($S_t=S_r=5\%$), the minimum observed jump angle is $21^\circ$ and the maximum jump angle is $76^\circ$ (for upstream Froude numbers between 3.8 and 4.5), whereas Eq. (2) predicts a minimum $\beta$ angle between 12 and $15^\circ$ and a maximum $\beta$ angle of around $70^\circ$ for the same inlet Froude numbers. Compared with the experiments, Eq. (2) slightly underestimates the minimum $\beta$ angle for small deflection angles and overestimates the measured maximum angle, as can be seen in Fig. 9. This might be partly due to uncertainties in the experimental measurements—the toe of the oblique jump was measured instead of the center of the jump (Fig. 7), which leads to a difference of a few degrees.

The exact location of the normal hydraulic jump in the minor flow channel in the Type II regime for the configurations $S_t=S_r=5\%$ and $S_t=5\%, S_r=1\%$ varies with the main flow discharge; the normal hydraulic jump in the minor channel moves upstream, away from the junction, discharge in the main channel increases (Fig. 18). It is interesting to note that for the configuration $S_t=5\%, S_r=1\%$, the symmetrical flow pattern (transition between Type I and Type III) occurs for $M_x/M_t \sim 0.42$ (Fig. 18). When $M_x/M_t=0.5$, the flow pattern is not symmetrical, but a normal hydraulic jump occurs in the inlet channel with the smaller slope.

**Type III**

For the Type III regime, two oblique hydraulic jumps form within the junction, one on each side of the deflection line. Both upstream flows reach the junction in supercritical conditions, are
deflected as they cross the oblique jumps, and are then deflected towards the outlet channels (Fig. 19). It should be noticed that this regime requires very high Froude numbers, so it occurs relatively rarely and is very difficult to find in nature or even in manmade channels or street networks. Indeed, to enable the deflection of the flows by oblique jumps, both deflection angles $\theta$ and $\theta'$ must be smaller than the maximum deflections $\theta_{\text{max}}$ and $\theta'_{\text{max}}$, respectively, as given by Eq. (2). With two inlet channels of equal slopes, such a situation can occur only if $\theta_{\text{max}} > 45^\circ$ and $\theta'_{\text{max}} > 45^\circ$, i.e., with $F_{x\text{,i}} > 5$ and $F_{y\text{,i}} > 5$ (Fig. 8). As already mentioned, it was necessary to modify the experimental facility [see Fig. 10(b)] in order to obtain the Type III flow configuration shown in Fig. 20 where $F_{x\text{,i}} = 5.45$ and $F_{y\text{,i}} = 5.32$.

Between the upstream corner $A$ and the middle of the diagonal, straight oblique jumps are observed with a deflection $\theta = 43^\circ$ for $Q_{y\text{,i}}$, and $\theta = 90^\circ - \theta = 47^\circ$ for $Q_{x\text{,i}}$. Measurements of $\beta$ confirm the validity of the concept of an “equivalent angled side wall,” as proposed by Behlke and Pritchett (1966) and this therefore justifies the use of $M_x/M_y$ instead of $W_x/W_y$ to account for the mutual deflection of the incoming flows. The measured values of jump angle are $\beta = 56^\circ$ and $\beta' = 62^\circ$, while the corresponding theoretical values given by Eq. (2) are $\beta_{\text{theoretical}} = 57^\circ$ and $\beta'_{\text{theoretical}} = 68^\circ$.

Fig. 19. Flow structure and flow pattern for Type III flow regime

Fig. 20. Measured water depth field for Type III regime

The stagnation zone and the deflection of the flow occurring near the downstream corner $C$ impose a transition to the subcritical regime, through a detached hydraulic jump located at the mid-diagonal, and the flow transported by the horizontal downstream branch in the modified installation is smaller than that measured for large downstream slopes. Consequently, backwater effects influence the flow near corners $B$ and $D$, and create semicircular jumps, similar to the radial jump that forms when a jet of water impacts on a flat plate. These jumps join the jump detached from $C$, forming a semicircular boundary between the subcritical and the supercritical zones, that was not observed with downstream slopes $S_x = S_y = 5\%$ for the intermediate Type II–Type III flow.

Intermediate Flow Configurations

These configurations correspond to the presence of at least one detached hydraulic jump in the junction, similar in form to a bow wave. This phenomenon appears in one of the incoming channels when the deflection imposed by the other inflow is slightly higher than the maximum deflection allowed through an oblique jump. A detached hydraulic jump then forms in the incoming channel; the normal part of the jump is located just upstream of the intersection while the oblique part of the jump crosses the entry section and is located partly in the upstream branch, partly in the intersection (e.g., Fig. 21). When the imposed deflection increases further, the detachment length also increases. The whole bow jump is translated upstream; the oblique part of the jump intersects the wall of the incoming channel; the confinement then changes the bow jump into a straight, normal hydraulic jump, perpendicular to the channel walls. The intermediate configurations for the flow Types I, II, and III must be considered individually, because if there is a detached jump the incoming flow is supercritical in one part of the entry section and subcritical in the other. The downstream flow development in the intersection is thus also different. An example of a configuration intermediate between Types I and III is presented in Fig. 21. As $\theta > \theta_{\text{max}}$ and $\theta' > \theta'_{\text{max}}$, the hydraulic jumps cannot be oblique jumps located in the junction, but the jumps are not sufficiently detached to be confined within the inlet channels. As described above, both hydraulic jumps start within the upstream channels near corner $A$, several centimeters from the junction entry sections and they end...
within the junction. Fig. 22 shows a configuration intermediate between a Type II and a Type III flow. The hydraulic jump facing $Q_xi$ upstream flow is an oblique jump, whereas the hydraulic jump facing $Q_yi$ upstream flow is a detached jump, mostly located in the intersection and almost parallel to the junction entry section.

**Additional Flow Details**

**Recirculating Zone**

On the upstream side of both downstream channels, a recirculating zone appears close to the junction. In this area the water depth is usually much lower than in the rest of the intersection, especially for Types II and III flow patterns (Figs. 12, 14, 16, and 20) and it may even be totally dry. A comparison of the size of the recirculating zones for the three subregimes of Type II (Figs. 12, 14, and 16) shows that as the deflection angle $\theta$ decreases, the width and length of the recirculating zone in the minor output channel $Q_{yo}$ increases and the width and length of the recirculating zone in the other channel $Q_{xo}$ decreases.

**Wave Fronts in Downstream Channels**

The incoming flows impact on each other, increasing the water depth in the crossing as shown in Fig. 16. This increase is advected into an outlet branch, creating a standing wave called a “deflection wave.” As the deflection wave front reaches the wall of the downstream channel, it is reflected and the wave may then oscillate back and forth over a long distance downstream from the junction. Similarly, the oblique jump is also reflected from the downstream channel walls. If the oblique jump angle $\beta$ is smaller than 45°, the oblique jump and the deflection wave reach the same downstream branch $x$ and only one wave reflection will be visible, as already reported by Greated (1968) and Hager (1989). On the other hand, for higher values of $\beta$, the oblique jump reflections occur in the other downstream branch. For instance for the Type II subregime 3 presented in Fig. 23 where $\theta < 45^\circ$ and $\beta > 45^\circ$, the deflection wave is reflected from the downstream walls of the main channel and the oblique jump is reflected from the walls of the side channel downstream of the intersection, and both wave reflections remain visible until the end of the channels.

**Eddying Regions**

Eddying zones, characterized by a large increase in water depth, are caused by two phenomena: the impact of the deflection wave on the downstream channel walls (Figs. 12, 14, and 16) and the stagnation zone in the close vicinity of the junction corner $C$ (Figs. 20–22), where the flow can also rise suddenly and even break (Fig. 24). The presence of these eddies usually increases dramatically the risks associated with a real urban flood. Indeed, at the intersection, the local increase of water depth may submerge a building or a wall, or may reach windows much higher than the average water level in the junction.

Another zone of high water depth can be identified, located on the deflection line but near the upstream corner (Figs. 12, 14, 16, and 20–22). This may be due to the fact that the minor subcritical...
flow is not yet perfectly deflected from its original axis at the upstream limit of the deflection line, whereas it is already deflected at its downstream end.

Conclusions

Three main flow patterns, called Types I, II, and III, can form in supercritical flow in a four-channel junction and are separated by three intermediate flow configurations. The main differences between the three patterns are the location and nature of the hydraulic jumps which depend on the deflection angles values \( \theta \) and \( \theta' \) and the maximum deflection angles for which oblique jumps can form (Ippen 1951). Experimental observations show that the Type II regime can be split into three subregimes depending on the angles of the specific structures occurring within the junction. For each flow pattern, we have measured the surface profile and we have developed a schematic description of the velocity field, emphasizing the recirculating zones, reflected waves, and eddying when they appear. Finally, we have suggested a relationship between the location of the hydraulic jumps and the ratio \( M_r/M_s \), but it is not clear whether this relationship can be generalized beyond the conditions studied in these experiments.

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Notation

The following symbols are used in this paper:

- \( b \) = channel width;
- \( F \) = Froude number;
- \( g \) = acceleration of gravity;
- \( h \) = flow depth;
- \( L \) = channel length;
- \( M \) = specific momentum;
- \( M_r \) = specific momentum ratio; \( M_r = M_t/M_s \);
- \( M_s \) = total specific momentum; \( M_t = M_s + M_r \);
- \( Q \) = discharge;
- \( S \) = channel slope;
- \( v \) = velocity;
- \( W \) = flow power;
- \( W_{rr} \) = flow power ratio; \( W_{rr} = W_r/W_s \);
- \( W_t \) = total flow power; \( W_t = W_s + W_r \);
- \( z \) = bottom elevation;
- \( \beta \) = oblique jump angle;
- \( \beta_{br} \) = oblique jump angle measured at bottom of jump;
- \( \beta_c \) = oblique jump angle measured at crest of jump;
- \( \beta_\text{theoretical} \) = oblique jump angle calculated using Eq. (2);
- \( \lambda \) = angle between entrance section in main channel and line joining upstream corner to intersection between depression wave and exit section in main channel (see Fig. 11, for example);
- \( \theta, \theta' \) = deflection angle of main and minor flow, respectively;
- \( \theta_{\text{max}}, \theta'_{\text{max}} \) = maximal flow deflection angles;
- \( \delta \) = junction angle; and
- \( \rho \) = fluid density.

Subscripts

- \( i \) = inlet;
- \( n \) = normal;
- \( o \) = outlet;
- \( r \) = ratio;
- \( x \) = main flow direction; and
- \( y \) = minor flow direction.

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